

Dynamics of QCD at large N_c *

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Abstract

Dynamics of confinement, chiral symmetry breaking and thermal phase transition is considered at large N_c . It is argued that these phenomena are quantitatively well described within the Gaussian stochastic model of the QCD vacuum. Selfconsistent equations are written for the field correlators of the model, yielding important connection between gluonic correlation length of the vacuum and the string tension. Comparison to other approaches and experimental and lattice data is given.

1 Introduction

Large N_c limit first introduced for QCD in [1] has two implications. First, it allows to neglect nonplanar perturbative (P) diagrams and reduce nonperturbative (NP) background diagrams to simple expressions; second, it allows to establish hierarchy of physical characteristics, which can be compared to experiment. E.g. the decay widths of all mesons Γ_n are $O(1/N_c)$ while masses M_n are $O(N_c^0)$, in experiment on average $\Gamma_n/M_n \sim 0.1$ which gives an idea of the parameter for the real QCD. Also white objects do not interact in the leading N_c order, which leads to the so-called topological expansion of high-energy scattering amplitudes. All dynamical picture of QCD at large

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N_c looks selfconsistent and realistic, and there have been many attempts to derive it from the first principles [2].

Recently a new and universal method was suggested in QCD [3] which allows to calculate all amplitudes in terms of a set of basic quantities: field correlators (FC). The simplest approximation uses only the lowest FC, quadratic in field strength, and was called the Gaussian Stochastic Model (GSM). Both lattice calculations [4] and theoretical estimates [5] show that accuracy of GSM is of the same order as of the large N_c expansion, and that GSM yields the dominant contribution to the most QCD phenomena in the leading large N_c order.

The dynamical input of GSM – the quadratic FC – is usually taken in a simple exponential form from the lattice data [4]. In the most recent paper [6] the large N_c limit was used to obtain selfcoupled equations for FC, which in principle allow for selfconsistent determination of FC from the QCD Lagrangian. Keeping only the lowest FC in these equations one is able to connect NP parameters (string tension, gluon condensate, correlation length) between themselves and to the P parameter (Λ_{QCD}). In this way there appears a realistic possibility to have a unified consistent picture of QCD dynamics at large N_c , described by the only scale parameter as it should be. In this talk we discuss confinement (section 2), chiral symmetry breaking (CSB) (section 3), thermal phase transition (section 4), with concluding remarks in section 5.

2 Confinement at large N_c

Lattice calculations performed at $R \lesssim 1.5fm$ confirm that static fundamental quarks are confined by the linear potential $V(R) = \sigma R$ [7] even in the presence of dynamical fermions [8] which in principle should screen the quark charges at large distances. Since dynamical quarks break the fundamental string in the next $0(1/N_c)$ order this means that in the region $R \lesssim 1.5fm$ the fundamental string does not break and the leading $0(N_c^0)$ approximation is valid.

Another feature of the confinement, also seen in lattice calculations at $R \lesssim 1.5fm$ [7] is the so-called Casimir scaling [9], namely that the string tension $\sigma(j)$ for static charges in the j representation of the color group

$SU(N_c)$ obey the law

$$\frac{\sigma(j)}{\sigma(fund)} = \frac{C_2(j)}{C_2(fund)}, \quad (1)$$

where $C_2(j)$ is the quadratic Casimir operator,

$$C_2(adj, N_c) = N_c, \quad C_2(fund, N_c) = \frac{N_c^2 - 1}{2N_c}. \quad (2)$$

The breaking of the adjoint string is the $0(1/N_c^2)$ process which happens at large distance $R_{br}(N_c)$ which grows with N_c , as can be seen from the expression [9]

$$\langle W_{adj}(C) \rangle = C_1 \exp(-\sigma_{adj} RT) + \frac{C_2}{N_c^2} \exp(-V_s(R)T). \quad (3)$$

Here $V_s(R)$ is the screened quark potential.

We now show that all the above features are nicely described in the GSM. To this end one writes the nonabelian Stokes theorem [5,10]

$$\begin{aligned} \langle W(C) \rangle &= \langle \frac{1}{N_C} \text{Ptr} \exp ig \int_c A_\mu dx_\mu \rangle = \\ &= \frac{1}{N_c} \langle P \text{tr} \exp ig \int_S d\sigma_{\mu\nu} F_{\mu\nu}(u, z_0) \rangle \end{aligned} \quad (4)$$

where we have defined

$$F_{\mu\nu}(u, z_0) = \Phi(z_0, u) F_{\mu\nu}(u) \Phi(u, z_0), \quad \Phi(x, y) = P \exp ig \int_y^x A_\mu dz_\mu \quad (5)$$

and integration in (4) is over the surface S inside the contour C , while z_0 is an arbitrary point, on which $\langle W(C) \rangle$ evidently does not depend. In the Abelian case the parallel transporters $\Phi(z_0, u)$ and $\Phi(u, z_0)$ cancel and one obtains the usual Stokes theorem.

Note that the nonabelian Stokes theorem, eq. (4), is gauge invariant even before averaging over all vacuum configurations – the latter is implied by the angular brackets in (4).

One can now use the cluster expansion theorem to express the r.h.s. of (4) in terms of FC , namely [3,5]

$$\langle W(C) \rangle = \frac{\text{tr}}{N_C} \exp \sum_{n=1}^{\infty} \frac{(ig)^n}{n!} \int d\sigma(1) d\sigma(2) \dots d\sigma(n) \ll F(1) \dots F(n) \gg \quad (6)$$

where lower indices of $d\sigma_{\mu\nu}$ and $F_{\mu\nu}$ are suppressed and $F(k) \equiv F_{\mu_k\nu_k}(u^{(k)}, z_0)$.

Note an important simplification – the averages $\ll F(1)...F(n) \gg$ in the color symmetric vacuum are proportional to the unit matrix in color space, and the ordering operator P is not needed any more.

Eq. (6) expresses Wilson loop in terms of gauge invariant FC , also called cumulants, defined in terms of FC as follows:

$$\ll F(1)F(2) \gg = \langle F(1)F(2) \rangle - \langle F(1) \rangle \langle F(2) \rangle \quad (7)$$

$$\begin{aligned} \ll F(1)F(2)F(3) \gg &= \langle F(1)F(2)F(3) \rangle - \ll F(1)F(2) \gg \langle F(3) \rangle - \\ &- \langle F(1) \rangle \ll F(2)F(3) \gg - \langle F(2) \rangle \ll F(1)F(3) \gg - \langle F(1) \rangle \langle F(2) \rangle \langle F(3) \rangle \end{aligned}$$

In the lowest approximation, which corresponds to the GSM, one keeps only the quadratic in F term, namely

$$D_{\mu\nu\lambda\sigma} \equiv \frac{1}{N_c} \text{tr} \langle F_{\mu\nu}(x) \Phi(x, y) F_{\lambda\sigma}(y) \Phi(y, x) \rangle \quad (8)$$

The form (8) has a general decomposition in terms of two Lorentz scalar functions $D(x-y)$ and $D_1(x-y)$ [3]

$$\begin{aligned} D_{\mu\nu\lambda\sigma} &= (\delta_{\mu\lambda}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\lambda}) \mathcal{D}(x-y) + \\ &+ \frac{1}{2} \partial_\mu \{ [(h_\lambda \cdot \delta_{\nu\sigma} - h_\sigma \delta_{\nu\lambda}) + \dots] \mathcal{D}_1(x-y) \} \end{aligned} \quad (9)$$

Here the ellipsis implies terms obtained by permutation of indices. It is important that the second term on the r.h.s. of (9) is a full derivative by construction.

Insertion of (9) into (6) yields the area law of Wilson loop with the string tension σ

$$\begin{aligned} \langle W(C) \rangle &= \exp(-\sigma S_{min}) \\ \sigma &= \frac{1}{2} \int D(x) d^2x (1 + O(FT_g^2)) \end{aligned} \quad (10)$$

where $O(FT_g^2)$ stands for the contribution of higher cumulants, and S_{min} is the minimal area for contour C .

The string tension for heavy (static) quarks is an infinite sum of FC of the field $F_{14} \equiv E_1$ integrated over the plane 14:

$$\sigma \sim \sum_n \frac{g^n}{n!} \prod_i^{n-1} d^2r_i \ll E_1(0) E_1(r_1) E_1(r_1 + r_2) \dots E_1(\sum r) \gg$$

One can identify parameter of expansion in the sum above to be (only even powers of n enter the sum)

$$\zeta = (\bar{E}_1 T_g^2)^2$$

where T_g is the gluonic correlation length in the vacuum and $\bar{E}_1^2 \cong g^2 < (E_1^a)^2 \geq \frac{4\pi^2}{12} G_2 \approx (0.2 GeV)^2$ where G_2 is the standard gluon condensate. Naively one would expect that the correlation length T_g is of the order of confinement radius R_c , $T_g \sim R_c \sim \Lambda_{QCD}^{-1}$, in which case since $\bar{E}_1 \sim \Lambda_{QCD}^2$ the parameter ζ is $\zeta \sim 1$ and all FC are equally important (that would be true for ordinary FC, but connected FC may have additional small parameter at large n due to cancellation in (7)).

However lattice calculations confirm that T_g is much smaller [4], indeed $T_g \approx 0.2 \div 0.3 fm$ and therefore parameter ζ is small

$$\zeta = 0.04 \div 0.1$$

The regime $\zeta \ll 1$, which seems to be characteristic for real QCD, can be called the regime of the weak confinement. In this case the dynamics of quarks and gluons is adequately described in most cases by the lowest (Gaussian) correlator [3,4].

Note that D_1 does not enter σ , but gives rise to the perimeter term and higher order curvature terms. On the other hand the lowest order perturbative QCD contributes to D_1 and not to D , namely the one-gluon-exchange contribution is

$$D_1^{pert}(x) = \frac{16\alpha_s}{3\pi x^4} \quad (11)$$

Nonperturbative parts of $D(x)$ and $D_1(x)$ have been computed on the lattice [4] using the cooling method, which suppresses perturbative fluctuations. As one can see both functions are well described by an exponent in the measured region, and $D_1(x) \sim \frac{1}{3}D(x) \sim \exp(-x/T_g)$, where $T_g \sim 0.2 fm$.

The string tension (10) can be computed from the lattice data [4] extrapolated to small distances, and agrees within 10-20% with the standard value $\sigma \approx 0.2 GeV^2$. Hence the Gaussian correlator alone can explain the string tension.

We conclude this chapter with discussion of confinement for charges in higher representations. As it was stated in the previous chapter, our definition of confinement based on lattice data, predicts the linear potential between static charges in any representation, with string tension proportional to the quadratic Casimir operator.

Consider therefore the Wilson loop (4) for the charge in some representation; the latter was not specified above in all eqs. leading to (10). One can write in general

$$A_\mu(x) = A_\mu^a T^a, \text{tr}(T^a T^b) = \frac{1}{2} \delta^{ab} \quad (12)$$

Similarly to (6) one has for the representation $j = (m_1, m_2, \dots)$ of the group $SU(N)$ with dimension $N(j)$

$$\langle W(C) \rangle = \frac{1}{N(j)} \text{tr}_j \exp \sum_{n=1}^{\infty} \frac{(ig)^n}{n!} \int d\sigma(1) \dots d\sigma(n) \ll F(1) \dots F(n) \gg \quad (13)$$

and by the usual arguments one has Eq.(9) .

Due to the color neutrality of the vacuum each cumulant is proportional to the unit matrix in the color space, e.g. for the lowest cumulant one has

$$\begin{aligned} \langle F(1)F(2) \rangle_{ab} &= \langle F^c(1)F^d(2) \rangle T_{an}^c T_{nb}^d = \\ &= \langle F^e(1)F^e(2) \rangle \frac{1}{N_c^2 - 1} T_{an}^c T_{nb}^c = \Lambda^{(2)} C_2(j) \cdot \hat{1}_{ab}, \end{aligned} \quad (14)$$

where we have used the definition

$$T^c T^c = C_2(j) \hat{1} \quad (15)$$

and introduced a constant not depending on representation,

$$\Lambda^{(2)} \equiv \frac{1}{N_C^2 - 1} \langle F^e(1)F^e(2) \rangle, \quad (16)$$

and also used the color neutrality of the vacuum,

$$\langle F^c(1)F^d(2) \rangle = \delta_{cd} \frac{\langle F^e(1)F^e(2) \rangle}{N_C^2 - 1} \quad (17)$$

For the next – quartic cumulant one has

$$\begin{aligned} \ll F(1)F(2)F(3)F(4) \gg_{\alpha\epsilon} &= \ll F^{a_1}(1)F^{a_2}(2)F^{a_3}(3)F^{a_4}(4) \gg \times \\ &\times T_{\alpha\beta}^{a_1} T_{\beta\gamma}^{a_2} T_{\gamma\delta}^{a_3} T_{\delta\epsilon}^{a_4} = \Lambda_1^{(4)} (C_2(j))^2 \delta_{\alpha\epsilon} + \Lambda_2^{(4)} (T^{a_1} T^{a_2} T^{a_1} T^{a_2})_{\alpha\epsilon} \end{aligned} \quad (18)$$

Thus one can see in the quartic cumulant a higher order of quadratic Casimir and higher Casimir operators.

The string tension for the representation j is the coefficient of the diagonal element in (14) and (18)

$$\sigma(j) = C_2(j) \int \frac{g^2 \Lambda^{(2)}}{42} d^2 x + O(C_2^2(j)) \quad (19)$$

where the term $O(C_2^2(j))$ contains higher degrees of $C_2(j)$ and higher Casimir operators.

Comparing our result (19) with lattice data [7] one can see that the first quadratic cumulant should be dominant as it ensures proportionality of $\sigma(j)$ to the quadratic Casimir operator.

Thus we see that GSM yields a simple confinement picture which is consistent with lattice data and large N_c considerations at least in the region $R \lesssim 1.5 fm$. At larger R and fixed N_c and for adjoint charges the screening occurs which needs higher cumulants, and this happens in the $O(1/N_c^2)$ order.

3 CSB at large N_c

In the large N_c limit the phenomenon of CSB is supported by the Coleman–Witten theorem [11], whereas at $N_c = 2, 3$ lattice data show clearly CSB in the order parameters of quark condensate $\langle \bar{\psi}(0)\psi(0) \rangle$, which is nonzero for $T \leq T_c$. Also parity doublets are missing in the hadronic spectrum. There is still another important feature of CSB seen in the heavy–light $q\bar{Q}$ system, namely the scalar confining interaction for the light quark, which clearly signals CSB. In the present section we shall present results of recent studies of this system [12,13], which is the simplest from the dynamical point of view.

We start with the quark Green’s function and write the effective quark Lagrangian in presence of a static source, using the averaging of the partition function over gluonic field A_μ .

To take into account the static source we consider the generalized coordinate gauge [10] and express A_μ through $F_{\mu\nu}$ as

$$A_\mu(x) = \int_C ds \frac{dz_\alpha(s, x)}{ds} F_{\alpha\beta}(z) \frac{dz_\beta}{dx_\mu} \quad (20)$$

where the contour C starts at x_μ and is described by $z_\mu(x, s)$ (in the usual coordinate gauge $z_\mu(x, s) = sx_\mu, 0 \leq s \leq 1$). The effective Lagrangian is (a

more extended version of this derivation see in [12]).

$$\begin{aligned}\mathcal{L}_{eff}(\psi^+\psi) &= \int \psi^+(x)(-i\hat{\partial} - im)\psi(x)d^4x + \\ \frac{1}{2N_c} \int d^4x d^4y &(\psi_a^+(x)\gamma_\mu\psi_b(x))(\psi_b^+(y)\gamma_{\mu'}\psi_a(y)) \times \\ &\times J_{\mu\mu'}(x, y)\end{aligned}\tag{21}$$

where we have defined

$$\begin{aligned}J_{\mu\mu'}(z, w) &= \int_C^z du_\alpha \int_C^w dv_\gamma (\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}) \frac{du_\beta}{dz_\mu} \frac{dv_\delta}{dw_{\mu'}} \times \\ &\times D(u - v)\end{aligned}\tag{22}$$

and $D(u)$ is defined in (9) yielding the string tension

$$\sigma = \frac{1}{2} \int_{-\infty}^{\infty} d^2u D(u)\tag{23}$$

Note that we have neglected in (21) higher field correlators, which were argued above to yield subdominant contribution.

The Lagrangian (21) can be used to obtain equations for the quark Green's function S in the large N_c limit, where the following rule of replacement holds

$$\psi_b(x)\psi_b^+(y) \rightarrow \langle \psi_b(x)\psi_b^+(y) \rangle = N_c S(x, y),\tag{24}$$

One obtains a system of equations for the quark Green's function S and the mass operator M

$$iM(z, w) = J_{\mu\nu}(z, w)\gamma_\mu S(z, w)\gamma_\nu\tag{25}$$

$$(-i\hat{\partial}_z - im)S(z, w) - i \int M(z, z')S(z', w)d^4z' = \delta^{(4)}(z - w)\tag{26}$$

The system of equations (25,26) is exact in the large N_c limit, when higher correlators are neglected and defines unambiguously both the interaction kernel M and the Green's function S . One should stress at this point again that both S and M are not the one-particle operators but rather two-particle operators, with the role of the second particle played by the static source. It is due to this property, that S and M are gauge invariant operators, which is very important to take confinement into account properly. Had we worked

with one-particle operators, as is the habit in QED and sometimes also in QCD, then we would immediately loose the gauge invariance and the string, and hence confinement.

The CSB can manifest itself in solutions of (25,26) in several ways. One is the appearance of the nonzero chiral condensate

$$\langle \bar{\psi}(0)\psi(0) \rangle = iN_c \text{tr} S(0,0) \quad (27)$$

This was estimated using the relativistic WKB method in [12] to be

$$\langle \bar{\psi}(o)\psi(0) \rangle \sim -N_c f\left(\frac{\sigma}{T_g}, \sigma^{3/2}\right) \quad (28)$$

where T_g is the gluonic correlation length [3,4].

Another manifestation of CSB is the scalar confinement, which is seen at large distances r . Indeed one write expansion for the Green's function S and M in the inverse powers of the string mass $M_{str} = \sigma r + m$, and the time-averaged Green's function \bar{S} satisfies an equation [13]:

$$[-i\vec{\gamma}\vec{\partial} - i(m + \sigma|\vec{z}|)]\bar{S}(\vec{z}, \vec{w}) = \delta^{(3)}(\vec{z} - \vec{w}) \quad (29)$$

In (29) the string term σr enters as a scalar, which signals CSB.

Equations (28) and (29) exemplify the connection between CSB and confinement in the large N_c limit. The necessary appearance of CSB in this limit was proved earlier in [11] but no hint to the possible mechanism was given. Here we demonstrate that CSB occurs due to the string formation (which is contained in factor σ in (28), (29) and the kernel J in (25),(26)), but this effect comes to the existence only due to the solution of nonlinear equations (25),(26). One should mention that these equations in contrast to those of NJL, are nonlocal and therefore do not need cut-off. The CSB solution is not obtained by perturbation expansion of (25),(26), but rather is an extra, nonperturbative solution existing due to the nonlinearity. A similar situation occurs in the NJL model.

4 Phase transition at large N_c

To describe the phase transition in QCD we shall use the basic idea, suggested in [14], that the confining phase is governed by the NP fields, while in the deconfining phase one has P fields in the NP background of magnetic fields.

Therefore one should first introduce the background formalism where both P and NP fields enter.

We derive here basic formulas for the partition function, free energy and Green's function in the NP background formalism at $T > 0$ [15]. The total gluonic field A_μ is split into a perturbative part a_μ and NP background B_μ

$$A_\mu = B_\mu + a_\mu \quad (30)$$

where both B_μ and a_μ are subject to periodic boundary conditions. The principle of this separation is immaterial for our purposes here, and one can average over fields B_μ and a_μ independently using the 'tHooft's identity¹

$$Z = \int DA_\mu \exp(-S(A)) = \frac{\int DB_\mu \eta(B) \int Da_\mu \exp(-S(B+a))}{\int DB_\mu \eta(B)} \quad (31)$$

$$\equiv \langle\langle \exp(-S(B+a)) \rangle_a \rangle_B$$

with arbitrary weight $\eta(B)$. In our case we choose $\eta(B)$ to fix field correlators and string tension at their observed values.

The partition function can be written as

$$Z(V, T, \mu = 0) = \langle Z(B) \rangle_B ,$$

$$Z(B) = N \int D\phi \exp(-\int_0^\beta d\tau \int d^3x L(x, \tau)) \quad (32)$$

where ϕ denotes all set of fields a_μ, Ψ, Ψ^+, N is a normalization constant, and the sign $\langle \rangle_B$ means some averaging over (nonperturbative) background fields B_μ .

The inverse gluon propagator in the background gauge is

$$W_{\mu\nu}^{ab} = -D^2(B)_{ab} \cdot \delta_{\mu\nu} - 2gF_{\mu\nu}^c(B)f^{acb} \quad (33)$$

where

$$(D_\lambda)_{ca} = \partial_\lambda \delta_{ca} - igT_{ca}^b B_\lambda^b \equiv \partial_\lambda \delta_{ca} - gf_{bca} B_\lambda^b \quad (34)$$

Integration over ghost and gluon degrees of freedom in (32) yields

$$Z(B) = N' (det W(B))_{reg}^{-1/2} [det(-D_\mu(B)D_\mu(B+a))]_{a=\frac{\delta}{\delta J}} \times$$

$$\times \{1 + \sum_{l=1}^{\infty} \frac{S_{int}^l}{l!} (a = \frac{\delta}{\delta J})\} \exp(-\frac{1}{2} J W^{-1} J)_{J_\mu = D_\mu(B)F_{\mu\nu}(B)} \quad (35)$$

¹private communication to the author, December 1993.

One can consider strong background fields, so that gB_μ is large (as compared to Λ_{QCD}^2), while $\alpha_s = \frac{g^2}{4\pi}$ in that strong background is small at all distances.

In this case Eq. (35) is a perturbative sum in powers of g^n , arising from expansion in $(ga_\mu)^n$.

In what follows we shall discuss the Feynman graphs for the free energy $F(T)$, connected to $Z(B)$ via

$$F(T) = -T \ln \langle Z(B) \rangle_B \quad (36)$$

We are now in position to make expansion of Z and F in powers of ga_μ (i.e. perturbative expansion in α_s), and the leading-nonperturbative term Z_0, F_0 – can be represented as a sum of contributions with different N_c behaviour of which we systematically will keep the leading terms $0(N_c^2), 0(N_c)$ and $0(N_c^0)$.

To describe the temperature phase transition one should specify phases and compute free energy. For the confining phase to lowest order in α_s free energy is given by Eq.(36) plus contribution of energy density ε at zero temperature

$$F(1) = \varepsilon V_3 - \frac{\pi^2}{30} V_3 T^4 - T \sum_s \frac{V_3 (2m_s T)^{3/2}}{8\pi^{3/2}} e^{-m_s/T} + 0(1/N_c) \quad (37)$$

where ε is defined by scale anomaly [16]

$$\varepsilon \simeq -\frac{11}{3} N_c \frac{\alpha_s}{32\pi} \langle (F_{\mu\nu}^a(B))^2 \rangle \quad (38)$$

and the next terms in (37) correspond to the contribution of mesons (we keep only pion gas) and glueballs. Note that $\varepsilon = 0(N_c^2)$ while two other terms in (37) are $0(N_c^0)$.

For the second phase (to be the high temperature phase) we make an assumption that there all color magnetic field correlators are the same as in the first phase, while all color electric fields vanish. Since at $T = 0$ color-magnetic correlators (CMC) and color-electric correlators (CEC) are equal due to the Euclidean $0(4)$ invariance, one has

$$\langle (F_{\mu\nu}^a(B))^2 \rangle = \langle (F_{\mu\nu}^a)^2 \rangle_{el} + \langle (F_{\mu\nu}^a)^2 \rangle_{magn}; \quad \langle F^2 \rangle_{magn} = \langle F^2 \rangle_{el} \quad (39)$$

The string tension σ which characterizes confinement is due to the electric fields [5], e.g. in the plane (i4)

$$\sigma = \sigma_E = \frac{g^2}{2} \int \int d^2x \langle \text{tr} E_i(x) \Phi(x, 0) E_i(0) \phi(0, x) \rangle + \dots \quad (40)$$

where dots imply higher order terms in E_i .

Vanishing of σ_E liberates gluons and quarks, which will contribute to the free energy in the deconfined phase their closed loop terms with all possible windings. As a result one has for the high-temperature phase (phase 2).

$$F(2) = \frac{1}{2}\varepsilon V_3 - (N_c^2 - 1)V_3 \frac{T^4 \pi^2}{45} - \frac{7\pi^2}{180} N_c V_3 T^4 n_f + 0(N_c^0) \quad (41)$$

Comparing (37) and (41), $F(1) = F(2)$ at $T = T_c$, one finds in the order $0(N_c)$, disregarding all meson and glueball contributions

$$T_c = \left(\frac{\frac{11}{3} N_c \frac{\alpha_s \langle F^2 \rangle}{32\pi}}{\frac{2\pi^2}{45} (N_c^2 - 1) + \frac{7\pi^2}{90} N_c n_f} \right)^{1/4} \quad (42)$$

For standard value of $G_2 \equiv \frac{\alpha_s}{\pi} \langle F^2 \rangle = 0.012 \text{ GeV}^4$ (note that for $n_f = 0$ one should use approximately 3 times larger value of G_2) one has for $SU(3)$ and different values of $n_f = 0, 2, 4$ respectively $T_c = 240, 150, 134$ MeV. This should be compared with lattice data [16] $T_c(\text{lattice}) = 240, 146, 131$ MeV. Agreement is quite good. Note that at large N_c one has $T_c = 0(N_c^0)$ i.e. the resulting value of T_c doesn't depend on N_c in this limit. Hadron contributions to T_c are $0(N_c^{-2})$ and therefore suppressed if T_c is below the Hagedorn temperature as it typically happens in string theory estimates.

5 Conclusion

In all examples considered above gluon correlators and condensates entered as a given input. It is important to find equations for FC which define them up to an overall scale to Λ_{QCD} . These equations have been suggested in [6] and are derived on the same physical basis as for the quark Green's function in section 3. Namely one assumes that a gluon is propagating in the nonperturbative background, described by FC, and one obtains equations for the gluon Green's function (in the field of the static charge, so that the Green's function is gauge-invariant, as in the case of the quark in section 3).

$$(-\partial_\lambda^2 \delta_{\mu\rho} + \partial_\mu \partial_\rho) G_{\rho\nu}(x, y) + \int M_{\mu\rho}^{(g)}(x, z) G_{\rho\nu}(z, y) d^4 z = \delta^{(4)}(x - y), \quad (43)$$

where the mass operator $M^{(g)}$ is approximately equal at large distances to $M^{(2,2)}$, where

$$M_{\mu\nu}^{(2,2)}(x, y) = \frac{N_c}{C_2^f} \delta^{(4)}(x - y) [J_{\lambda\lambda}(x, y) \delta_{\mu\nu} - J_{\mu\nu}(x, y)], \quad (44)$$

For more extended treatment and derivation of equations (47),(48) the reader is referred to [6].

The kernel $J_{\lambda\mu}$ in (44) is expressed through the field correlator $\langle FF \rangle$. To make equations selfconsistent one should express the latter through the gluon Green's function G . This is possible since one can always refer the color indices in each term of $\sum_{a=1}^{N_c^2-1} \langle F^a F^a \rangle$ to the group of fields b_μ . Then one can write symbolically

$$tr \langle F(x)F(y) \rangle \sim \partial_\mu \partial_\nu G(x, y) + (G(x, y))^2 + perm + ... \quad (45)$$

where ellipsis stands for higher cumulants.

The system of equations (43-44) allows for a nonperturbative solution, which violates the scale invariance present in the equations. This solution is defined by fixing one nonperturbative scale, e.g. the string tension σ . Then equations (43-44) predict that i) both field correlators $D(x), D_1(x)$ [3] exponentially decay at large x ; $D(x), D_1(x) \sim \exp(-x/T_g)$ in agreement with lattice data [4], and ii) the gluon correlation length T_g is connected to σ as [6]

$$1/T_g = (2.33)^{3/4} \sqrt{\frac{9\sigma}{2\pi}} \quad (46)$$

Insertion of the standard value $\sigma \approx 0.2 GeV^2$ yields $T_g \approx 0.2 fm$, which is in good agreement with lattice data [4].

We have derived equations for the gluon and quark propagators in the field of a static source. These equations possess symmetry (chiral for the quark and scale invariance for the gluon) which is violated by the nonperturbative solutions. One obtains in this way the CSB due to the confining kernel, and the confining kernel itself satisfies nonlinear equations. Properties of this kernel are in agreement with lattice data.

Thus the Gaussian model using the simplest FC can describe both qualitatively and quantitatively the basic QCD phenomena at large N_c .

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